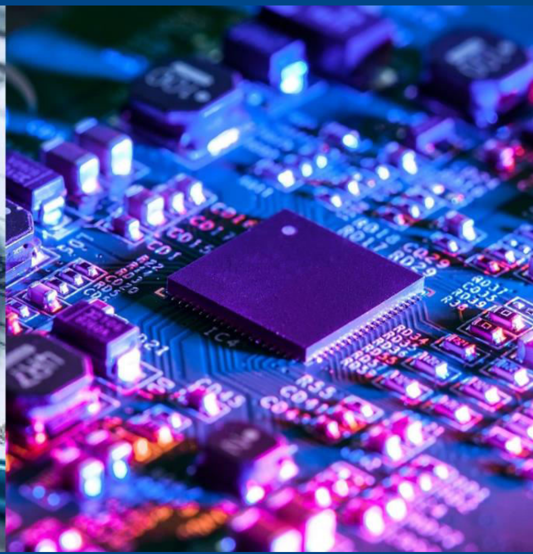
ISSN: 2582-7219





International Journal of Multidisciplinary Research in Science, Engineering and Technology

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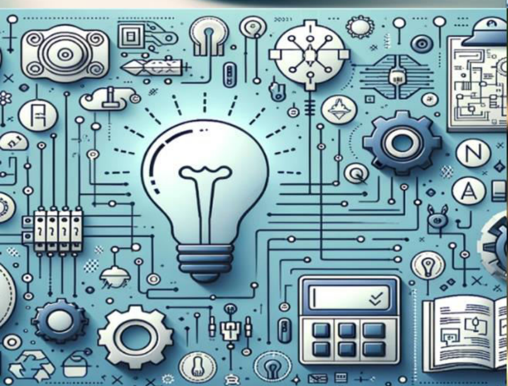

Impact Factor: 9.864

Volume 9, Issue 7, July 2026

ISSN: 2582-7219 | www.ijmrset.com | Impact Factor: 9.864| ESTD Year: 2018|

International Journal of Multidisciplinary Research in Science, Engineering and Technology (IJMRSET)

(A Monthly, Peer Reviewed, Refereed, Scholarly Indexed, Open Access Journal)

Partial Product Process under Replacement Policy N in an Alternative Repair Time

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ABSTRACT: In this paper, we study an partial product process under replacement policy N in an alternative repair time. This paper examines the replacement model and partial product process for a decaying system under various circumstances. An alternative model known as the Negligible Or Non-Negligible (NONN) repair time Using the partial product process under replacement policy N. Explicit expressions for the long run average cost under this model is developed. Existence of optimality conditions are deduced. Finally, conclusion is given.

KEYWORDS: Partial Product Process, Replacement Policy N in an NONN Repair Times

AMS subject classification: 60K10, 90B25

I. INTRODUCTION

An partial product processes are frequently used in contemporary manufacturing systems to increase production efficiency and lower operating costs. However, the system is frequently subject to unexpected failure or extreme shocks, such as equipment malfunctions, shortages of spare parts, or shifts in demand. The study of replacement models for repairable systems is one of the most fundamental and important topics in classical reliability theory. Lam Yeh (1988) introduced the geometric processes and replacement problem. Govindaraju, Rizwan and Thangaraj (2011) have studied an extreme shock maintenance model. Babu, Govindaraju and Rizwan (2018) introduced and studied replacement models where the consecutive repair time follow an increasing partial product process. Raajpandiyan, Syed Tahir Hussainy and Rizwan (2022) have studied optimal replacement models under partial product process.

An partial product process under Replacement Policy N in an Alternative Repair Time model with NONN (Negligible or Non-Negligible) repair times is examined in this work. The threshold of a deadly shock is monotone rather than constant, and the repair takes negligible time with probability p and non-negligible time with probability (1-p), making it a unique model. Furthermore, an expanding partial process is formed by the sucessive repair times following failure.

The rest of the paper is organized as follows. In Section II, we give general preliminaries. In Section III, we given model assumptions. In Section IV, we derive explicit expressions for the long-run average cost per unit time for this model under partial product process with Replacement Policy N using the alternative repair model. Finally, a conclusion in given section V.

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II. PRELIMINARIES

We require the following definitions.

Definition 2.1. (Barlow and Proschan, 1965) "A random variable X is said to be Stochastically Smaller than anther random variable Y, if $P(X > \alpha) \leq P(Y > \alpha)$ for all real α. It is denoted by $X \leq_{st} Y$."

Definition 2.2. (Babu, Govindaraju and Rizwan, 2018) "Let $\{X_n, n = 1, 2, 3, ...\}$ be a sequence of independent and non-negative random variables and let F(X) be the distribution function of X_1. Then $\{X_n, n = 1, 2, 3, ...\}$ is called Partial Product Process, if the distribution function of X_{i+1} is $F(\alpha_i X)$ $(i = 1, 2, 3, ...)$ where $\alpha_i > 0$ are real constants and $\alpha_i = \alpha_0 \alpha_1 \alpha_2 ... \alpha_{i-1}$."

Definition 2.3. "A stochastic process $\{X_n, n = 1, 2, 3, ...\}$ is said to be Stochastically increasing (decreasing) if $X_n \leq_{st} (\geq_{st}) X_{n+1}$ for all $n = 1, 2, 3, ...$"

Definition 2.4. "A partial product process is called a decreasing partial product process, if $\alpha_0 > 1$ and is called an increasing partial product process, if $0 < \alpha_0 < 1$."

Definition 2.5. (Renewal process) "If the sequence of nonnegative random variable $\{X_1, X_2, X_3, ...\}$ is independent and identically distributed, then the counting process $\{N(t), t \geq 0\}$ is said to be a renewal process."

Definition 2.6. (The T Policy) "It is a policy under which the system will be replaced whenever the working age of the system reaches T."

Definition 2.7. (The N Policy) "It is a policy under which we replace the system at the time of the N^{th} failure since the last replacement."

Remark 2.1. It is clear that if $\alpha_0 = 1$, then the partial product process is a renewal process.

Remark 2.2. let $E(Y_1) = \mu$, $var(Y_1) = \sigma^2$. Then for $j = 1,2,3,\cdots$,
$E(Y_{j+1}) = \frac{\mu}{\beta_0^{2j-1}}$ and $var(Y_{j+1}) = \frac{\sigma^2}{\beta_0^{2j}}$, where $\beta_0 > 0$.

Theorem 2.1 (Wald's equation) If $X_1, X_2, X_3, \cdots$ are independent and identically distribution random variables having finite expectations and if N is the stopping time for $X_1, X_2, \cdots$ such that $E[N] < \infty$, then

$$E\left[\sum_{n=1}^{N} X_n\right] = E[N]E[X_1]$$

Theorem 2.2 (Wald's equation for partial product process) Suppose that $\{Y_n, n = 1,2,3,\cdots\}$ forms a partial product process with ratio β_0 and $E[Y_1] = \mu < \infty$, then for $t > 0$, we have

$$E[V_{\omega(t)+1}] = \mu E\left[1 + \sum_{j=2}^{\omega(t)+1} \frac{1}{\beta_0^{2j-2}}\right]$$

where $\omega(t)$ is the counting process which represents the number of occurrences of an event up to time t.

Definition 2.8. (Thangaraj and Rizwan, 2001) "If a repair to a system after failure is done in negligible or non-negligible time, then it will be called a model with Negligible Or Non Negligible repair times." In this case, whenever the system fails,

© 2026 IJMRSET | Volume 9, Issue 7, July 2026| DOI:10.15680/IJMRSET.2026.0907007

ISSN: 2582-7219 | www.ijmrset.com | Impact Factor: 9.864| ESTD Year: 2018|

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two possibilities may arise: either, the repair takes Negligible time with probability p; or Non-Negligible time with probability 1 - p.

III. THE REPLACEMENT POLICY N

Here, we explore an Partial Product Process under Replacement Policy N in an Alternative Repair Time. The problem is to find an optimal replacement policy N* under the replacement policy N, minimizing the long run average cost. We consider the following assumptions on the model of a deterministic degenerative repairable system under shock.

Assumption 3.1. At the beginning, a new simple repairable system is installed. Whenever the system fails, it will be repaired or replaced. The system will be replaced by an identical new one some times later.

Assumption 3.2. Let X_1 be the system's operating time after installation. Let X_n, n = 2, 3, ... be the operating times of the system after the (n - 1)-st repair in a cycle. The distribution of X_n is indicated by $F_n(.)$.

Assumption 3.3. After the initial failure, let ξ_1 represent the repair time and G(y) represent the distribution function of ξ_1. Assume that $E(\xi_1) = \mu \geq 0$. When $\mu = 0$, it indicates that the anticipated repair time is very small. After (j + 1)-st failure, let ξ_{j+1} be the repair time for j = 1, 2, 3, ... and $G(\beta_0^{j-1} y)$ be the distribution function of ξ_{j+1}, where $0 < \beta_0 \leq 1$ is a constant and $E(\xi_{j+1}) = \frac{\mu}{\beta_0^{2^{j-1}}}$ for j = 1, 2, 3, ...The sequential repair durations from an increasing partial product process are $\{\xi_j, j = 1, 2, 3, ...\}$. By assumption,

$$E(\xi_n) = E(Y_n)\, P(Y_n \geq 0) + 1\ \ P(Y_n = 0)$$
$$= \frac{\mu}{\beta_0^{2^{n-1}}}\,(1 - p) + p$$

Assumption 3.4. The survival times (X_i), the NONN repair times (ξ_i) and the replacement time Z, are independent for i = 1, 2, ...

Assumption 3.5. The reward rate is r while the system is in operation. The replacement cost is R and the repair cost rate is c.

Assumption 3.6. The random variable Z, whose expected value is $E(Z) = \tau$, is the replacement time. If a replacement is done, a cycle is completed. A cycle is, in fact, the period between two succeeding replacements. So, it represents a renewal process with consecutive replacements. Finally, the consecutive cycle together with this cost incurred during each cycle forming a renewal reward process. According to the renewal reward theorem, the long run average cost per unit time under the replacement policy N is deduced in the following section.

IV. THE REPLACEMENT POLICY N WITH NONN REPAIR TIME

A policy for replacement according to a policy known as N policy with NONN repair time, the system is replaced at the N-th failure. The Problem is to determine an optimal replacement policy N that minimizes the long-run average cost per unit time. Thus a renewal process is formed by the sequence N_n, n = 1, 2, ... By the renewal reward theorem, the long-run average cost per unit time under the replacement policy N is given by:

$$\mathcal{C}(\mathrm{T}) = \frac{\text{the expected cost incurred in a cycle}}{\text{the expected length of a cycle}}$$
$$= \frac{E\left(\sum_{j=1}^{N-1} c\xi_j\right) - E\left(\sum_{i=1}^{N} rX_i\right) + R}{E\left(\sum_{i=1}^{N} X_i\right) + E\left(\sum_{j=1}^{N-1} \xi_j\right) + E(Z)}$$
$$= \frac{c\left(\sum_{j=1}^{N-1} E(\xi_j)\right) - r\left(\sum_{i=1}^{N} E(X_i)\right) + R}{\sum_{i=1}^{N} E(X_i) + \sum_{j=1}^{N-1} E(\xi_j) + E(Z)}$$

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$$= \frac{c\left(E(\xi_1) + \sum_{j=2}^{N-1} E(\xi_j)\right) - r\left(E(X_1) + \sum_{i=2}^{N} E(X_i)\right) + R}{E(X_1) + \sum_{i=2}^{N} E(X_i) + E(\xi_1) + \sum_{j=2}^{N-1} E(\xi_j) + \tau}$$

$$= \frac{c\left[\mu + \sum_{j=2}^{N-1}\left[\frac{\mu(1-p)}{\beta_0^{2^{j-1}}} + p\right]\right] - r\left[\lambda + \sum_{i=2}^{N}\frac{\lambda}{\beta_0^{2^{i-1}}}\right] + R}{\lambda + \sum_{i=2}^{N}\frac{\lambda}{\beta_0^{2^{i-1}}} + \mu + \sum_{j=2}^{N-1}\left[\frac{\mu(1-p)}{\beta_0^{2^{j-1}}} + p\right] + \tau}$$

$$= \frac{c\mu\left[1 + \sum_{j=2}^{N-1}\left[\frac{\mu(1-p)}{\beta_0^{2^{j-1}}} + p\right]\right] - \lambda r\left[1 + \sum_{i=2}^{N}\frac{1}{\beta_0^{2^{i-1}}}\right] + R}{\lambda\left[1 + \sum_{i=2}^{N}\frac{1}{\beta_0^{2^{i-1}}}\right] + \mu\left[1 + \sum_{j=2}^{N-1}\left[\frac{\mu(1-p)}{\beta_0^{2^{j-1}}} + p\right]\right] + \tau} - r + r$$

$$\mathcal{C}(N) = \frac{(c+r)\mu\left[1 + \sum_{j=2}^{N-1}\left[\frac{\mu(1-p)}{\beta_0^{2^{j-1}}} + p\right]\right] + R + r\tau}{\lambda\left[1 + \sum_{i=2}^{N}\frac{1}{\beta_0^{2^{i-1}}}\right] + \mu\left[1 + \sum_{j=2}^{N-1}\left[\frac{\mu(1-p)}{\beta_0^{2^{j-1}}} + p\right]\right] + \tau} - r$$

Thus, $\mathcal{C}(N) = D(N) - r$ where

$$D(N) = \frac{(c+r)\mu\left[1 + \sum_{j=2}^{N-1}\left[\frac{\mu(1-p)}{\beta_0^{2^{j-1}}} + p\right]\right] + R + r\tau}{\lambda\left[1 + \sum_{i=2}^{N}\frac{1}{\beta_0^{2^{i-1}}}\right] + \mu\left[1 + \sum_{j=2}^{N-1}\left[\frac{\mu(1-p)}{\beta_0^{2^{j-1}}} + p\right]\right] + \tau} - r$$

By minimizing $\mathcal{C}$ (N) or D(N), we can find the ideal replacement policy N* in NONN repair time.

V. CONCLUSION

In this paper, we have considered a partial product process in an alternative repair times maintenance model for a degenerative simple repairable system. An explicit expression for the long-run average cost under the replacement policy N with NONN repair times is derived. A numerical example is presented to demonstrate the theoretical results.

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